The XVA challenge.
An intuitive introduction
Part 1

Introduction: Pricing Challenges
Market developments

Spreads and swap curves January 2007 versus May 2016

- S45 Euro Swaps Curve
- S201 Euro (vs. 3M Euribor) Curve
- S232 Euro (vs. 1M Euribor) Swap Curve
- S133 Eonia Curve
Market developments

The (confidence) crisis

BNP Paribas announces credit difficulties, suspends redemptions from three money market funds, because of valuation difficulties in subprime assets. (August 8, 2007)

Collapse of Lehman Brothers (September 15, 2008)
Market developments

The (confidence) crisis
Market developments

Cross-currency basis spreads between EUR and USD

10YR XCCY basis EUR / USD
Pricing challenges

*Multiple curves and basis risks*

**Pre-crisis valuation**
- Tenor curve

**Post-crisis valuation**
- Tenor curve
- Overnight Index Swap (OIS) curve
- Collateral implied curve
- Break clauses (termination amount)
- Credit Value Adjustment (CVA)
- Debt Value Adjustment (DVA)
- Funding Cost Adjustment (FCA)
- Funding Benefit Adjustment (FBA)
- Collateral Value Adjustment (ColVA)
- Replacement Value Adjustment (RVA)
- Margin Value Adjustment (MVA)
- Capital Value Adjustment (KVA)
Part 2

Funding Costs, Funding Strategies
By Burgard & Kjaer
(Extended by Green, Kenyon (&Dennis))
This article looks at funding strategies in terms of holding or issuing own bonds. Reasonability of this assumption is much debated in the industry.

Strategy hedges out some but not all cashflows at own default.

Economic value to shareholders is calculated assuming that they disregard any remaining post default cash flows and pre-default balance sheet effects. I.e. M&M does not hold!

DEF: Funding Cost Adjustment: discounted expected value of post-default cashflows.

Assume one or two-way credit support annexes (CSA).
General semi-replication and pricing PDE

*Framework*

- Consider generic derivative contract, possibly collateralized, between issuer **B** and counterparty **C** with an economic value $\hat{V}$.

- Economic value $\hat{V}$ incorporates:
  - Risk of default of counterparty.
  - Risk of default of issuer.
  - Any net funding costs the issuer may encounter prior to own default.

- The (risk-free) Black Scholes price is denoted by $V$ and is denoted by the partial differential equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

- $\hat{V} - V = U$
- $U \equiv \text{CVA} + \text{DVA} + \text{FCA} + \text{COLVA}$. 
General semi-replication and pricing PDE

Framework

- Tradable instruments:
  - Counterparty zero coupon, zero recovery bond $P_c$. This bond accrues interest at $r_c$.
  - Issuers’ own bonds $P_1$ and $P_2$ of different seniority ($P_1$ junior, $P_2$ senior). Recovery rates $R_1$ and $R_2$ with $R_1 < R_2$ and $r_1 > r_2$.
  - Market instrument $S$ to hedge out market factors.
General semi-replication and pricing PDE

Framework

**General semi-replication strategy**

*We will describe a general semi-replication strategy that the issuer can deploy to perfectly hedge out market factors and counterparty default but which may not provide a perfect hedge in the event of the issuer's own" default.*
General semi-replication and pricing PDE

Tradable instruments and dynamics

• Assume the following dynamics for the instruments \((i = 1, 2)\):

\[
\begin{align*}
    dS &= \mu S dt + \sigma S dW \\
    dP_C &= r_C P_C^- dt - P_C^- dJ_C \\
    dP_i &= r_i P_i^- dt - (1 - R_i) P_i^- dJ_B
\end{align*}
\]

where \(J_B\) and \(J_C\) are default indicators for issuer B and counterparty C with

\[
J_B, J_C = \{0, 1\}
\]

With \(J = 1\) if there is a default for the respective party and 0 when this is not the case

• \(P_x^-\) is the pre-default price of bond \(x\)
For simplicity, assume zero basis, thus no basis risk, between bonds of different seniority:

\[ r_i - r = (1 - R_i)\lambda_B, \]

Where \( r \) the risk-free rate.

\( \lambda_B \) the spread of a zero-recovery, zero-coupon bond of the issuer.

Assumption of zero basis in order to express Value Adjustments in spreads on zero-coupon bonds of the issuer.
General semi-replication and pricing PDE

Boundary conditions

• \( \hat{V}(t, S, J_B, J_C) \) is the total economic value of the derivative to the issuer.

• General boundary conditions at default of the issuer or counterparty:
  \( \hat{V}(t, S, 1, 0) = g_B(M_B, X) \) if \( B \) defaults first.
  \( \hat{V}(t, S, 0, 1) = g_C(M_C, X) \) if \( C \) defaults first.

• With general close-out amounts \( M_B, M_C \) and collateral \( X \). These boundary conditions hold if \( M_B = M_C = V \), where \( V \) is the classic Black Scholes price (no default risks or funding costs).
General semi-replication and pricing PDE

Boundary conditions

- The following regular bilateral boundary conditions hold with collateral:

\[
\begin{align*}
  g_B &= (V - X)^+ + R_B (V - X)^- + X \\
  g_C &= R_C (V - X)^+ + (V - X)^- + X.
\end{align*}
\]

- Example:
  - \( R_B = R_C = 0.5 \)
  - \( V = 60, \ X = 50 \)

  - \( g_B = 10 + 0 + 50 = 60 \)
  - \( g_C = 0.5 \cdot 10 + 0 + 50 = 55 \)

- Regular boundary conditions are the following without collateral:

\[
\begin{align*}
  g_B &= V^+ + R_B V^- \quad \text{and} \quad g_C = R_C V^+ + V^-.
\end{align*}
\]
General semi-replication and pricing PDE

Set up of the Hedge Portfolio

- Set up a hedge portfolio $\Pi$ as:

$$\Pi(t) = \delta(t)S(t) + \alpha_1(t)P_1(t) + \alpha_2(t)P_2(t) + \alpha_C(t)P_C(t) + \beta_S(t) + \beta_C(t) - X(t).$$

- $\delta(t)$ is the number of units $S$.
- $\alpha_i(t)$ units of bonds $P_i$.
- $X(t)$ the collateral account (fully re-hypothecable) paying rate $r_X$.
- $\beta_S(t)$ the cash accounts used to finance $S$.
- $\beta_C(t)$ the cash accounts used to finance $P_C$.

- Lets skip for convenience of notation the time subscript $(t)$
General semi-replication and pricing PDE

Set up of the Hedge Portfolio

- Hence

$$\alpha_C P_C + \beta_C = 0 \quad \delta S + \beta_S = 0$$

- Which pay net rates of respectively $q_s - \gamma_s$ and $q_c$ with:
  - $\gamma_s$ is the yield on $S$.
  - $q_s$ is the financing rate with which $\beta_S$ is collateralized,
  - $q_c$ is the repo rate of the of the repo transaction with which the counterparty bond is assumed to be set up.
General semi-replication and pricing PDE

Funding Constraint

- Strategy is designed such that $\hat{V} + \Pi = 0$, except, possibly, at issuer default.

- $\alpha_1 P_1$ and $\alpha_2 P_2$ are used to finance or invest the remaining cash not funded via collateral.

- This results in the following funding constraint:

$$\hat{V} - X + \alpha_1 P_1 + \alpha_2 P_2 = 0.$$
General semi-replication and pricing PDE

Evolution of the Hedge Portfolio (1/3)

• The evolution of the hedge portfolio:

\[ d\Pi = \delta dS + \alpha_1 dP_1 + \alpha_2 dP_2 + \alpha_C dP_C(t) + d\beta_S + d\beta_C - d\bar{X}, \]

• The bar indicates a change in cash and collateral accounts excluding the rebalancing.
• \( \beta_S \) is collateralized with financing rate \( q_s \) and yield \( \gamma_S \).
• Counterparty bond assumed be setup up via a repo transaction costing rate \( q_s \).
• The derivatives collateral account is assumed to cost a collateral rate \( r_X \).
• This yields:

\[ d\beta_S = \delta(\gamma_S - q_S)Sdt, \]
\[ d\beta_C = -\alpha_C q_C P_Cdt, \]
\[ d\bar{X} = -r_X X dt. \]

\[ P \equiv \alpha_1 P_1 + \alpha_2 P_2 \]
\[ P_D \equiv R_1 \alpha_1 P_1 + R_2 \alpha_2 P_2 \]
General semi-replication and pricing PDE

Evolution of the Hedge Portfolio (2/3)

• Inserting dynamics of own bonds $dP_i$, dynamics of market factors $dS$, the change in cash $d\beta_S$, $d\beta_C$ and collateral $dX$ in the evolution of the hedge portfolio gives:

$$d\Pi = \left( r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda C \alpha C P_C + (\gamma - q) \delta S - r_X X \right) dt + (P_D - P)dJ_B - \alpha C P_C dJ_C + \delta dS$$
General semi-replication and pricing PDE

Evolution of the Hedge Portfolio (3/3)

• Inserting dynamics of own bonds \( dP_i \), dynamics of market factors \( dS \), the change in cash \( d\beta_s, d\beta_c \) and collateral \( dx \) in the evolution of the hedge portfolio gives:

\[
\begin{align*}
\bar{d}\Pi &= (r_1\alpha_1 P_1 + r_2\alpha_2 P_2 + \lambda C \alpha C P_C + (\gamma - q)\delta S - r_X X) \, dt \\
&\quad + (P_D - P) dJ_B - \alpha C P_C dJ_C + \delta dS
\end{align*}
\]

Factors and derivation explained

• \( \lambda_c = r_c - q_c \), thus \( \lambda_c \) is defined as the spread of the yield of \( P_c \) over its repo rate, i.e. the financing rate of the counterparty default hedge position.
• First two terms come from SDE of \( P_1 \) and \( P_2 \).
• Third term comes from SDE \( P_c \) and definition \( d\beta_c \), since \( \alpha_c r_c P_c - \alpha_c q_c P_c = \lambda_c \alpha_c P_c \).
• Fourth term comes from definition of \( d\beta_s \).
• Fifth term comes from definition \( dx \).
• The term in front of \( dJ_b \) follows from the definition of \( dP_1, dP_2 \) and \( P_D \) and is equal to the difference between the pre and post default values of the positions in own bonds.
• The seventh term follows from the SDE of \( P_c \).
General semi-replication and pricing PDE

Ito’s Lemma (1/3)

- Lets apply Ito’s lemma for jump diffusions:

\[ df(t, S_t) = f_t(t, S_t)dt + f_s(t, S_t)dS_t + \frac{1}{2} f_{ss}(t, S_t)d[S]_t + \Delta f(t, S_t)dJ. \]

- With \( \Delta f(t, S_t) = f(t, S_t + K_t) - f(t, S_t) \) the difference between the pre and post jump values if there is a jump of size \( K \) at time \( t \).

- Standard case \( d[S]_t = \sigma^2 S_t^2 dt \).

- A jump process for own defaults and a process for defaults of the counterparty.
General semi-replication and pricing PDE

Ito’s Lemma (2/3)

• Applying Ito’s Lemma results in:

\[ d\hat{V} = \partial_t \hat{V} dt + \partial_S \hat{V} dS + \frac{1}{2} \sigma^2 S^2 \partial^2_S \hat{V} dt + \Delta\hat{V}_B dJ_B + \Delta\hat{V}_C dJ_C, \]

• Intuitively same as standard application of Ito’s lemma plus extra terms for the impact of defaults.
• In the previous slide it holds that:

\[
\Delta \hat{V}_B = \hat{V}(t, S, 1, 0) - \hat{V}(t, S, 0, 0) = g_B - \hat{V}, \\
\Delta \hat{V}_C = \hat{V}(t, S, 0, 1) - \hat{V}(t, S, 0, 0) = g_C - \hat{V},
\]

• These are the values if the respective bond issuer defaults minus the value if none of the parties defaults.
General semi-replication and pricing PDE

\[ d\hat{V} + d\Pi \]

- Together the expressions for the portfolio and the derivative yield:

\[
d\hat{V} + d\Pi = \left( \partial_t \hat{V} + \frac{1}{2} \sigma^2 S^2 \partial^2 S \hat{V} + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 
\right.
\]
\[
+ \lambda C \alpha C P C + (\gamma - q) \delta S - r X X \right) dt
\]
\[
+ (g_B + P_D - X) dJ_B + (\Delta V_C - \alpha C P C) dJ_C + (\delta + \partial S \hat{V}) dS,
\]

- Please note that combining \((P_D - P) dJ_B\) and \(\Delta \hat{V}_B dJ_B\) with \((\hat{V} + P - X = 0)\) and \(\Delta \hat{V}_B = g_B - \hat{V}\) gives \((g_B + P_D - X) dJ_B\)
General semi-replication and pricing PDE

\[ d\hat{V} + d\Pi = \left( \partial_t \hat{V} + \frac{1}{2} \sigma^2 S^2 \partial^2_S \hat{V} + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 ight. \\
+ \lambda_C \alpha_C P_C + (\gamma - q) \delta S - r_X X \bigg) dt \\
+ (g_B + P_D - X) dJ_B + (\Delta \hat{V}_C - \alpha_C P_C) dJ_C + (\delta + \partial_S \hat{V}) dS, \]

- Choosing smart values for the weights in our portfolio results in elimination, see the next slide.
  
  \((\gamma - q) \delta S\) changes due to substitution,
  
  where \((\gamma - q)\) is the effective financing rate of the market factor \(S\).

\[ \alpha_C P_C = \Delta V_C \]
\[ \delta = -\partial_S \hat{V} \]
General semi-replication and pricing PDE

\[ d\hat{V} + d\Pi = \left( \partial_t \hat{V} + \frac{1}{2} \sigma^2 S^2 \partial_S^2 \hat{V} + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 \right. \]
\[ + \lambda_C \alpha_C P_C + (\gamma - q) \delta S - r_X X \bigg) dt \]
\[ + (g_B + P_D - X) dJ_B + (\Delta \hat{V}_C - \alpha_C P_C) dJ_C + (\delta + \partial_S \hat{V}) dS, \]
\[ \alpha_C P_C = \Delta V_C \]
\[ \delta = -\partial_S \hat{V} \]

\[ d\hat{V} + d\Pi = \left( \partial_t \hat{V} + \mathcal{A}_t \hat{V} - r_X X + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda_C \Delta \hat{V}_C \right) dt \]
\[ + (g_B + P_D - X) dJ_B, \]
\[ \mathcal{A}_t V \equiv \frac{1}{2} \sigma^2 S^2 \partial_S^2 V + (q_S - \gamma_S) S \partial_S V \]
General semi-replication and pricing PDE

Substituting the hedging error (1/2)

- Substituting the hedging error \( \epsilon_h = (g_B + P_D - X) \), results in substitution of last term.

- Substituting \( \Delta \hat{V}_c = g_c - \hat{V} \), yields \(-\lambda_c \hat{V} \) and \( \lambda_c g_c \).

- Substituting \( P_D = \alpha_1 R_1 P_1 + \alpha_2 R_2 P_2 \), using \( \hat{V} - X + \alpha_1 P_1 + \alpha_2 P_2 = 0 \) and using that \( \lambda_B (1 - R_i) = r_i - r \), thus \(-\lambda_B R_i = r_i - r - \lambda_B \). It follows that:

\[
\lambda_B g_B - \epsilon_h \lambda_B = -\lambda_B (\alpha_1 R_1 P_1 + \alpha_2 R_2 P_2 - X) \\
= (r_1 - r - \lambda_B) \alpha_1 P_1 + (r_2 - r - \lambda_B) \alpha_2 P_2 + \lambda_B X
\]

- \( \lambda_B X - \lambda_B P_1 - \lambda_B P_2 = -\lambda_B \hat{V} \), due to the funding constraint.

- \(-\alpha r P_1 - \alpha r P_2 - rX = -r \hat{V} \), due to the funding constraint.
General semi-replication and pricing PDE

Substituting the hedging error (2/2)

• Lets define: \( s_X = r_X - r \).

\[
\begin{align*}
\frac{d\hat{V}}{dt} + \frac{d\Pi}{dt} &= \left( \partial_t \hat{V} + \mathcal{A}_t \hat{V} - r_X X + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda_C \Delta \hat{V}_C \right) \, dt \\
&\quad + (g_B + P_D - X) \, dJ_B,
\end{align*}
\]

\[
\mathcal{A}_t V \equiv \frac{1}{2} \sigma^2 S^2 \partial_S^2 V + (q_S - \gamma_s) S \partial_S V
\]

\[
\begin{align*}
\frac{d\hat{V}}{dt} + \frac{d\Pi}{dt} &= \left( \partial_t \hat{V} + \mathcal{A}_t \hat{V} - (r + \lambda_B + \lambda_C) \hat{V} - s_X X + \lambda_C g_C + \lambda_B g_B - \epsilon_h \lambda_B \right) \, dt \\
&\quad + \epsilon_h \, dJ_B.
\end{align*}
\]
General semi-replication and pricing PDE

*Self financing & drift term*

- Issuer wants the evolution of the total portfolio to evolve in a self financed manner, while alive.

- Therefore the drift term has to be zero so only the hedging error term is left, which yields the following:

\[
\partial_t \hat{V} + A_t \hat{V} - (r + \lambda_B + \lambda_C)\hat{V} = s_X X - \lambda_C g_C - \lambda_B g_B + \lambda_B \epsilon_h
\]

\[
\hat{V}(T, S) = H(S),
\]

- where \( H(S) \) is the payout of the derivative at maturity \( (T) \).
General semi-replication and pricing PDE

Correction to the Black-Scholes Price

• Interested in the correction $U$ with respect to BS price $V$, $U = \hat{V} - V$, gives:

$$
\partial_t U + \mathcal{A}_t U - (r + \lambda_B + \lambda_C)U = s_X X - \lambda_C (g_C - V) - \lambda_B (g_B - V) + \lambda_B \epsilon_h
$$

$$
U(T, S) = 0.
$$

• Use the definitions of $g_B$ and $g_c$, as $\hat{V}$ if respectively the issuer or the counterparty defaults.
General semi-replication and pricing PDE

Feynman-Kac application (1/4)

• The intuition to Feynman-Kac is that the drift term has to cancel out.

• Therefore integrated over time the first term on the LHS has to be equal to $f(x, t)$ discounted with the discount process.

• Thus the following holds: 

$$ (1 - \int_t^T (r + \lambda_B + \lambda_C)dr)U = \mathbb{E}^Q [\int_t^T f(x, t)dr]. $$

• The boundary condition follows from the no arbitrage condition.
General semi-replication and pricing PDE

Feynman-Kac application (2/4)

- We apply the Feynman-Kac theorem to:

\[
\partial_t U + A_t U - (r + \lambda_B + \lambda_C) U = s_X X - \lambda_C (g_C - V) - \lambda_B (g_B - V) + \lambda_B \epsilon_h \\
U(T, S) = 0.
\]

- Refresher, consider the following PDE

\[
\frac{\partial u}{\partial t}(x,t) + \mu(x,t) \frac{\partial u}{\partial x}(x,t) + \frac{1}{2} \sigma^2(x,t) \frac{\partial^2 u}{\partial x^2}(x,t) - V(x,t)u(x,t) + f(x,t) = 0,
\]

- Subject to boundary condition \( u(x, T) = \psi(x) \), then it holds that:

\[
u(x,t) = E^Q \left[ \int_t^T e^{-\int_t^\tau V(X_{t'}, \tau)} d\tau f(X_t, r) dr + e^{-\int_t^T V(X_{t'}, \tau) d\tau} \psi(X_T) \bigg| X_t = x \right]
\]

- Under probability measure \( Q \) such that: \( dX = \mu(X, t) dt + \sigma(X, t) dW^Q \).

- In our case \( \psi(X_T) = 0 \) and \( V(X, t) = (r + \lambda_B + \lambda_C) \):
General semi-replication and pricing PDE

Feynman-Kac application (3/4)

- The last slide yields the summation of the following terms.
- Minus term $f(x, t)$ is on the RHS in our case, thus the signs turn.
- Boundary condition $U(T, S)$ equal to zero, hence it cancels out.
- Discount process is defined as: $D_{\hat{r}(t,u)} = \exp \left( - \int_t^u \hat{r}(v) dv \right)$
- This gives:

$$\int_t^T - D_{r+\lambda_B+\lambda_c}(t,u) \cdot (\lambda_c(u)E[V(u) - g_c(V(u), X(u))] + \lambda_B(u)(E[V(u) - g_c(V(u), X(u))] + E[\epsilon_h(u)]) + s_x E[X(u)]) du.$$
General semi-replication and pricing PDE

Feynman-Kac application (4/4)

- Split the expression in the different value adjustments for the dynamics they describe: $U \equiv CVA + DVA + FCA + COLVA$.

\[
\begin{align*}
CVA &= - \int_t^T \lambda_C(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [V(u) - g_C(V(u), X(u))] \, du \\
DVA &= - \int_t^T \lambda_B(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [V(u) - g_B(V(u), X(u))] \, du \\
FCA &= - \int_t^T \lambda_B(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [\epsilon_h(u)] \, du \\
COLVA &= - \int_t^T s_X(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [X(u)] \, du,
\end{align*}
\]
General semi-replication and pricing PDE

Interpetations value adjustments

• The CVA is the discounted expected value of the difference between the BS value and the close out value when the counterparty defaults times the effective financing rate of the counterparty integrated over T.

• The DVA is computed in a similar manner but with the spread of a zero coupon zero recovery bond of the issuer and the close out amount of the issuer.

• The FCA is equal to the discounted expected value of the hedging error times the spread for the issuer integrated over time.

• The COLVA is equal to the discounted expected value of the collateral times the spread of the collateral integrated over time.
Examples

Perfect Replication

- In this example we assume that it is possible to hedge perfectly
- Hence it holds that: \( \epsilon_h = (g_B + P_D - X) = 0 \)
- The PDE then becomes:

\[
\partial_t \hat{V} + A_t \hat{V} - (r + \lambda_C + \lambda_B) \hat{V} = s_X X - \lambda_B g_B - \lambda_C g_C.
\]

- By the no windfall condition, \( g_B + \alpha_1 R_1 P_1 + \alpha_2 R_2 P_2 - X = 0 \), we get:
- \( \alpha_2 P_2 = (-\alpha_1 R_1 P_1 + X - g_B)/R_2 \) substituting this in the funding constraint and multiplying with \( R_2 \) gives:
- \( \hat{V} - X + \alpha_1 P_1 + (-\alpha_1 R_1 P_1 + X - g_B)/R_2 = 0 \),
- \( (R_2 - R_1) \alpha_1 P_1 = -R_2 \hat{V} + (R_2 - 1)X + g_B \),
- Moving the other terms to the RHS gives:

\[
\alpha_1 = \frac{R_2 \hat{V} - g_B + (1 - R_2)X}{(R_1 - R_2)P_1},
\]
Examples

Perfect Replication

- The adjustments for the regular bilateral boundary conditions with collateral become:

\[
CV\ = \ -(1 - R_C) \int_t^T \lambda_C(u)D_{r+\lambda_B+\lambda_C}(t,u)\mathbb{E}_t \left[ (V(u) - X(u))^+ \right] \, du
\]

\[
DV\ = \ -(1 - R_B) \int_t^T \lambda_B(u)D_{r+\lambda_B+\lambda_C}(t,u)\mathbb{E}_t \left[ (V(u) - X(u))^+ \right] \, du
\]

- \( FCA = 0 \) because \( \epsilon_h = 0 \)
- \( COLVA = 0 \) because \( r = r_x \)
- For the CVA it holds that:
  \[ V - g_c = V - R_c(V - X)^+ - (V - X)^- - X = (1 - R_c)(V - X)^+. \]
  If uncollateralized, thus \( X = 0 \), the expression becomes the classical CVA.
- For the DVA it holds that:
  \[ V - g_B = V - (V - X)^+ - R_B(V - X)^- - X = (1 - R_B)(V - X)^-. \]
Part 3

Margin Value Adjustment & Capital Value Adjustment
As a result of derivative transactions with its clients, banks are required to hold three different types of capital under Basel III (CRD IV, CRR):

01 **Market risk**

Capital against market movements (i.e. interest rate risk capital for an interest rate swap) that change the market value of the derivative. This results in market risk capital in the trading book or the banking book.

02 **Counterparty credit risk**

Capital for the risk that the client defaults and (part of) the current market value of the derivative is lost.

03 **Credit value adjustment**

Capital for the risk that the credit quality (e.g. credit rating, credit spread) of the client changes.

This capital results in value adjustments for the bank, the capital value adjustment (KVA).

- This means that banks start pricing capital costs – KVA – into their derivatives trades.
Margin value adjustment (MVA)

Costs of central clearing of derivatives

- **Definition of a derivative**
- The definition of a derivative is made in MiFID I, impacting other the legislation that relies on the MiFID definition of financial instrument, including EMIR and CRD IV.
- Furthermore, based on the Securities Markets Authority (ESMA), the Delegated Regulation of the EU requires the clearing of the following interest rate swap categories, which are detailed in Annex 1 to the Delegated Regulation:

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><strong>Fixed-to-Float (i.e. plain vanilla) interest rate swaps</strong> that:</td>
<td>• Reference the Euro Interbank Offered Rate (&quot;EURIBOR&quot;) or the London Interbank Offered Rate (&quot;LIBOR&quot;) • Have a maturity of 28 days to 50 years • Settled in one of the following currencies: EUR, GPB, USD or JPY</td>
</tr>
<tr>
<td><strong>Basis (i.e. Float-to-Float) interest rate swaps that:</strong></td>
<td>• Reference EURIBOR or LIBOR • Have a maturity of 28 days to 50 years • Settled in one of the following currencies: EUR, GPB, USD or JPY</td>
</tr>
<tr>
<td><strong>Forward Rate Agreements (‘FRAs’) that:</strong></td>
<td>• Reference EURIBOR or LIBOR • Have a maturity of three days to three years • Settled in one of the following currencies: EUR, GPB or USD</td>
</tr>
<tr>
<td><strong>Overnight Index Swaps (‘OIS’) that:</strong></td>
<td>• Reference Euro OverNight Index Average, FedFunds or the Sterling OverNight Index Average • Have a maturity of seven days to three years • Settled in one of the following currencies: EUR, GPB or USD</td>
</tr>
</tbody>
</table>

Margin value adjustment (MVA)
Extension Burgard & Kjaer with KVA and MVA

**Expressions value adjustments**

- Separating the value adjustments by the dynamics they describe yields:

\[
\text{CVA} = - (1 - R_C) \int_t^T \lambda_C(u) e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[(V(u))^+ \right] du \tag{33}
\]

\[
\text{DVA} = - (1 - R_B) \int_t^T \lambda_B(u) e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[(V(u))^+ \right] du \tag{34}
\]

\[
\text{FCA} = - (1 - R_B) \int_t^T \lambda_B(u) e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[(V(u))^+ \right] du \tag{35}
\]

\[
\text{COLVA} = - \int_t^T s_X(u) e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[X(u) \right] du
\]

\[
\text{KVA} = - \int_t^T e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[K(u)(\gamma_K(u) - r_B(u)\phi) \right] du \tag{36}
\]

\[
\text{MVA} = - \int_t^T ((1 - R_B)\lambda_B(u) - s_I(u)) e^{-\int_t^u (r(s) + \lambda_B(s) + \lambda_C(s)) ds} \mathbb{E}_t \left[I(u) \right] du \tag{37}
\]

- Please note that calculations involve nested MC simulations.
Appendix A

Semi-replication with single bond
Appendix A
Semi-replication with single bond

- Issuer uses a single own bond with recovery rate $R_F$
- Thus it holds that:
  \[ \alpha_1 P_1 = 0, \]
  \[ \alpha_2 P_2 = -(\hat{V} - X) = -(V + U - X), \]
- Define: $r_F = r + s_F$.
- If funding constraint is fulfilled no degrees of freedom to hedge own default
- Using $P_D = -R_F(\hat{V} - X)$ the hedging error becomes:
  \[ \epsilon_h = g_B + P_D - X = g_B - \hat{R}_F \hat{V} - (1 - \hat{R}_F)X, \]
- Insert the hedging error in:
  \[ \partial_t \hat{V} + A_t \hat{V} - \left( r + \lambda_B + \lambda_C \right) \hat{V} = s_X X - \lambda_C g_C - \lambda_B g_B + \lambda_B \epsilon_h \]
  \[ \hat{V}(T, S) = H(S), \]
- To give:
  \[ \partial_t \hat{V} + A_t \hat{V} - (r_F + \lambda_C) \hat{V} = -\lambda_C g_C(V, X) - (r_F - r_X)X \]
  \[ \hat{V}(T, S) = H(S) \]
Appendix A

Semi-replication with single bond

• The term $\lambda_B g_B$ drops out, using $(1 - R_i)\lambda_B = r_i - r$, gives $(r_F - r_X)X$ and the term $- r_F \hat{V}$ on the LHS. Replacing $\hat{V}$ with $U$ yields:

$$\partial_t U + A_t U - (r_F + \lambda_C)U = -\lambda_C(g_C(V, X) - V) + s_F(V - X) + s_X X$$

$$U(T, S) = 0$$

• Applying Feynman-Kac yields:

$$CVA_F = -(1 - R_C) \int_t^T \lambda_C(u) D_{r_F + \lambda_C}(t, u) \mathbb{E}_t [(V(u) - X)^+] \, du$$

$$DVA_F = -\int_t^T s_F(u) D_{r_F + \lambda_C}(t, u) \mathbb{E}_t [(V(u) - X(u)^-) \, du$$

$$FCAF = -\int_t^T s_F(u) D_{r_F + \lambda_C}(t, u) \mathbb{E}_t [(V(u) - X(u)^+) \, du$$

$$COLV A_F = -\int_t^T s_X(u) D_{r_F + \lambda_C}(t, u) \mathbb{E}_t [X(u)] \, du.$$ 

Combining $DVA_F$ and $FCAF$ into a $FVA_F$ gives:

$$FVA_F = DVA_F + FCAF = - \int_t^T s_F(u) D_{r_F + \lambda_C}(t, u) \mathbb{E}_t [V(u) - X(u)] \, du.$$
Appendix B

Semi-replication with no shortfall at own default
Appendix B

Semi-replication with no shortfall at own default

- This strategy does not aim to monetize on the potential windfall upon own default by entering into an offsetting spread position.
- This strategy does not generate shortfalls at own default.
- It results in the usual bilateral CVA adjustment plus a FCA term for regular close-outs, hence it is an extension of the existing framework.
- The strategy consists out of the following positions:
  - The investment in $p_1$ bonds should equal the difference between $\hat{V}$ and $V$.
  - Hold the investment in $p_2$ equal to the amount given by the funding constraint.
  - The adjustment $U$ falls away upon default and is funded via a zero recovery bond.

$$
\alpha_1 p_1 = -(\hat{V} - V) = -U,
\alpha_2 p_2 = -\alpha_1 p_1 - \hat{V} + X = -(V - X).
$$
Appendix B

Semi-replication with no shortfall at own default, hedging error

• The hedging error becomes, because $R_1 = 0$ and $R_2 = R_B$:
• $\epsilon_h = (V - X)^+ + R_B (V - X)^- + X + P_D - X = (V - X)^+ + R_B (V - X)^- - R_B (V - X) = (1 - R_B) (V - X)^+$.

$$\epsilon_h = (1 - R_B)(V - X)^+,$$

• This is always a windfall to the bondholders of the issuers.
Appendix B

Semi-replication with no shortfall at own default, value adjustments

- Applying Feynman-Kac in a similar matter as before yields:

\[ CVA = -(1 - R_C) \int_t^T \lambda_C(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [(V(u) - X(u))^+] \, du \]

\[ DVA = -(1 - R_B) \int_t^T \lambda_B(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [(V(u) - X(u))^+] \, du \]

\[ FCA = -(1 - R_B) \int_t^T \lambda_B(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [(V(u) - X(u))^+] \, du \]

\[ COLVA = - \int_t^T s_X(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [X(u)] \, du. \]

- We can rewrite the DVA and FCA as:

\[ FVA \equiv DVA + FCA = -(1 - R_B) \int_t^T \lambda_B(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [V(u) - X(u)] \, du. \]
Appendix B

Semi-replication with no shortfall at own default, one-way CSA

- Only the issuer posts collateral when the risk-free value is out of the money,
- Hence $X = V^-.$
- The adjustments become:

\[
CVA = -(1 - R_C) \int_t^T \lambda_C(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [V^+(u)] \, du
\]

(28)

\[
FCA = -(1 - R_B) \int_t^T \lambda_B(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [V^+(u)] \, du
\]

(29)

\[
COLVA = -\int_t^T s_X(u) D_r + \lambda_B + \lambda_C(t, u) \mathbb{E}_t [V^-(u)] \, du.
\]

(30)

- We can rewrite the DVA and FCA as:
Appendix B

Semi-replication with no shortfall at own default, set-offs

- A set-off is a legal agreement that allows the surviving party to settle outstanding derivative claims of the defaulting party with bonds of the defaulting party.
- For regular bilateral set-offs without collateral we get:
  - $g_B = R_B V$ and $g_c = R_C V$
- The value adjustments become:

\[
CV_A = -(1 - R_C) \int_t^T \lambda_C(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [V(u)] \, du
\]

\[
DV_A = -(1 - R_B) \int_t^T \lambda_B(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t [V(u)] \, du.
\]

- With the hedging error equal to zero the FCA vanishes.
Appendix C

Results & conclusion – Burgard & Kjaer
Appendix C

Results & conclusion – Burgard & Kjaer

• Different funding strategies for different issuers generate different funding costs when
the issuer is alive against different effects when he defaults

<table>
<thead>
<tr>
<th>Setup ((λ_B=100bp/500bp))</th>
<th>No CSA</th>
<th>One-way CSA</th>
<th>Set-off</th>
<th>No CSA</th>
<th>One-way CSA</th>
<th>Set-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL CVA</td>
<td>-467,283</td>
<td>-594,543</td>
<td>-283,227</td>
<td>61,393</td>
<td>-499,142</td>
<td>-435,111</td>
</tr>
<tr>
<td>Strategy I FCA (Issuer)</td>
<td>-198,024</td>
<td>-198,024</td>
<td>0</td>
<td>-832,560</td>
<td>-832,560</td>
<td>0</td>
</tr>
<tr>
<td>Strategy I FCA (Cparty)</td>
<td>382,081</td>
<td>0</td>
<td>0</td>
<td>336,056</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Strategy I valuation asymmetry</td>
<td>580,105</td>
<td>198,024</td>
<td>0</td>
<td>1,168,616</td>
<td>832,560</td>
<td>0</td>
</tr>
<tr>
<td>Strategy I Hedge error</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Strategy II FCA (Issuer)</td>
<td>-210,622</td>
<td>-212,309</td>
<td>-7,468</td>
<td>-916,177</td>
<td>-952,191</td>
<td>-61,743</td>
</tr>
<tr>
<td>Strategy II FCA (Cparty)</td>
<td>370,178</td>
<td>-32,650</td>
<td>-22,876</td>
<td>343,988</td>
<td>-26,229</td>
<td>-36,233</td>
</tr>
<tr>
<td>Strategy II valuation asymmetry</td>
<td>580,800</td>
<td>179,659</td>
<td>15,408</td>
<td>1,260,165</td>
<td>925,962</td>
<td>25,510</td>
</tr>
<tr>
<td>Strategy II Hedge error</td>
<td>271,162</td>
<td>322,741</td>
<td>116,278</td>
<td>341,913</td>
<td>580,533</td>
<td>198,742</td>
</tr>
</tbody>
</table>

Table 1: The FCA, hedge errors and valuation asymmetries for the OTM swap for the case of no-CSA, one-way CSA (issuer posts) and a set-off. In all the examples \(λ_C = 300bp\).
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